EE 330 Lecture 43

Digital Circuits

- Elmore Delay
- Power Dissipation

Fall 2024 Exam Schedule

Exam 1 Friday Sept 27

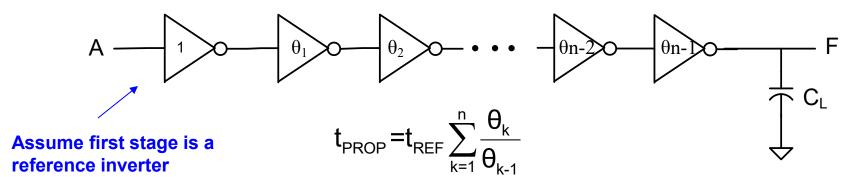
Exam 2 Friday October 25

Exam 3 Friday Nov 22

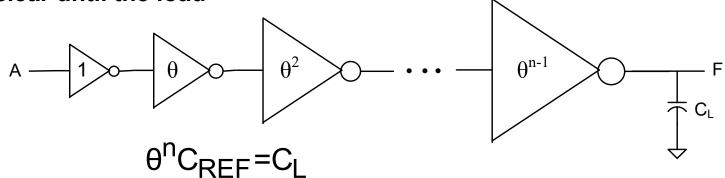
Final Exam Monday Dec 16 12:00 - 2:00 PM

Summary: Propagation Delay in Multiple-Levels of Logic with Stage Loading

)o— =	1/3		
	Equal Rise/Fall	Equal Rise/Fall (with OD)	Minimum Sized	Asymmetric OD (OD _{HL} , OD _{LH})
C_{IN}/C_{REF}				
Inverter	1	OD	1/2	OD _{HL} +3 • OD _{LH}
NOR	3k+1 4	3k+1 • OD	1/2	4 OD _{HL} +3k • OD _{LH}
NAND	$\frac{3+k}{4}$	3+k 4 OD	1/2	$k \bullet OD_{HL} + 3 \bullet OD_{LH}$
Overdrive				4
Inverter				
HL	1	OD	1	OD_HL
LH	1	OD	1/3	OD_LH
NOR HL	1	OD	1	OD_HL
LH	1	OD	1/(3k)	OD_LH
NAND HL	1	OD	1/k	OD_HL
LH	1	OD	1/3	OD_LH
t _{PROP} /t _{REF}	$\sum_{k=1}^{n} \mathbf{F}_{l(k+1)}$	$\sum_{k=1}^{n} \frac{F_{l(k+1)}}{OD_k}$	$\frac{1}{2} \sum_{k=1}^{n} \mathbf{F}_{I(k+1)} \left(\frac{1}{\mathbf{OD}_{HLk}} + \frac{1}{\mathbf{OD}_{LHk}} \right)$	$\frac{1}{2} \sum_{k=1}^n \textbf{F}_{\textbf{I}(k+1)} \Bigg(\frac{1}{\textbf{OD}_{\textbf{HLk}}} + \frac{\textbf{1}}{\textbf{OD}_{\textbf{LHk}}} \Bigg)^{4}$



Order reduction strategy: Assume overdrive of stages increases by the same factor clear until the load



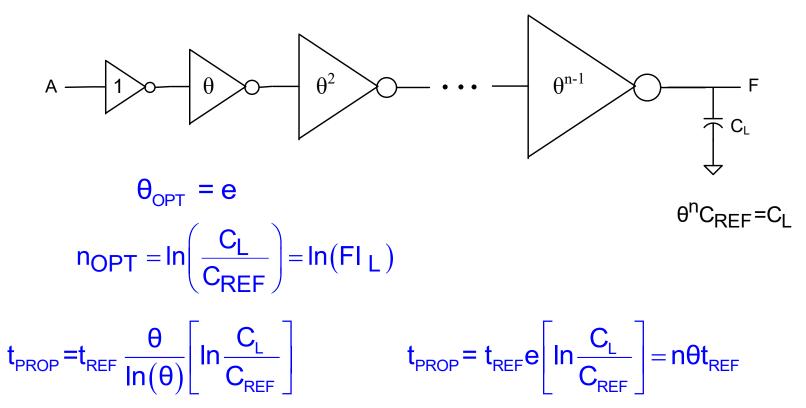
This becomes a 2-parameter optimization (minimization) problem!

Unknown parameters: $\{\theta, n\}$

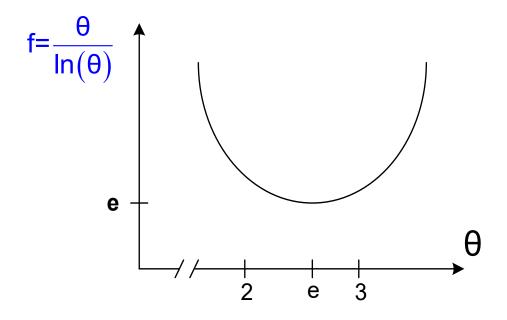
One constraint : $\theta^{n}C_{RFF}=C_{l}$



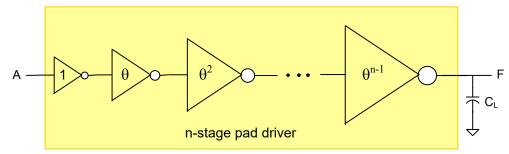
One degree of freedom



A practical solution



- minimum at θ =e but shallow inflection point for 2< θ <3
- practically pick θ =2, θ =2.5, or θ =3
- since optimization may provide non-integer for n, must pick close integer



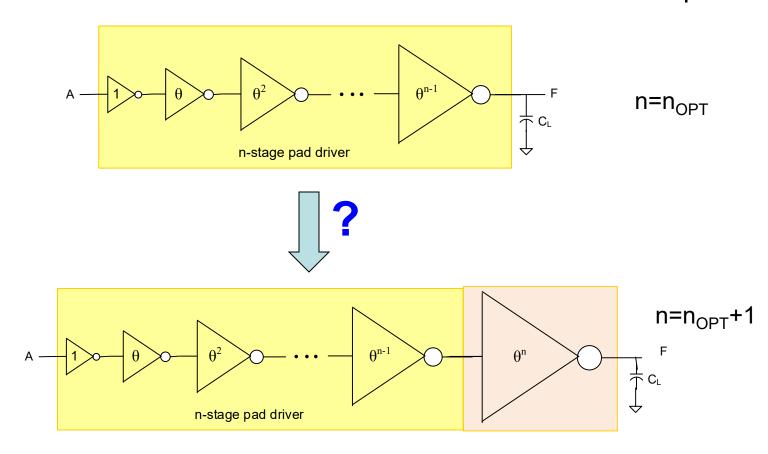
Example: Design a pad driver for driving a load capacitance of 10pF with equal rise/fall times, determine t_{PROP} for the pad driver, and compare this with the propagation delay for driving the pad with a minimum-sized reference inverter.

For
$$\theta = 2.5$$
, n=8 $W_{REF} = W_{MIN}$
 $W_{nk} = 2.5^{k-1} \cdot W_{REF}$, $W_{pk} = 3 \cdot 2.5^{k-1} \cdot W_{REF}$

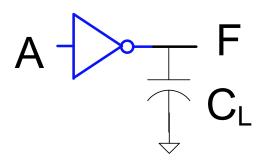
$$L_n = L_p = L_{MIN}$$

k	n-channel		p-channel	
1	1	VVMIN	3	VVMIN
2	2.5	VVMIN	7.5	VVMIN
3	6.25	VVMIN	18.75	VVMIN
4	15.6	VVMIN	46.9	VVMIN
5	39.1	VVMIN	117.2	VVMIN
6	97.7	VVMIN	293.0	VVMIN
7	244.1	VVMIN	732.4	VVMIN
8	610.4	VVMIN	1831.1	VVMIN

Will the circuit operate even faster if we increase the number of stages beyond n_{opt}?



Fundamental Limit on Size of Load that Can be Driven at given Clock Rate



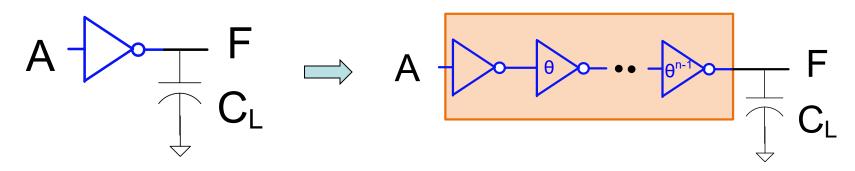
Can C_L=10pF be clocked at 100 MHz with a reference inverter?

Assume
$$C_{REF}=4fF$$
, $t_{REF}=20ps$ $f_{IN-MAX}=1/t_{PROP}$

$$\begin{split} t_{\text{PROP}} &= t_{\text{REF}} \bullet \text{FI}_{\text{LOAD}} = 20 p \, \text{sec} \bullet \frac{10 p F}{4 f F} = 20 p \, \text{sec} \bullet 2500 = 50 n \, \text{sec} \\ f_{\text{IN-MAX}} &= \frac{1}{50 n \, \text{sec}} = 20 \text{MHz} \end{split}$$

No! f_{IN-MAX} <100MHz

Fundamental Limit on Size of Load that Can be Driven at given Clock Rate



Can C₁ = 10pF be clocked at 100 MHz if a pad driver (sized for equal rise/fall) is used?

Assume
$$C_{REF}$$
=4fF, t_{REF} =20ps

$$f_{IN-MAX} = 1/t_{PROP}$$

$$f_{\text{IN-MAX}} = 1/t_{\text{PROP}}$$
 $FI_{\text{LOAD}} = \frac{10pF}{4fF} = 2500$

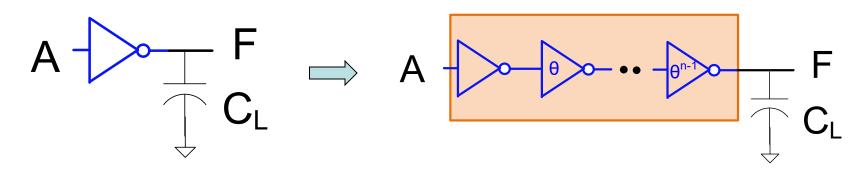
$$n_{OPT} = ln \left(\frac{C_L}{C_{REF}} \right) = ln(FI_L) = 7.8 \approx 8$$

$$t_{\text{PROP}}$$
= $n\theta t_{\text{REF}} = 8 \bullet e \bullet t_{\text{REF}} = 434 \text{ psec}$

$$f_{IN-MAX} = \frac{1}{n\theta t_{RFF}} = \frac{1}{434 \text{ psec}} = 2.30 \text{ GHz}$$

Yes!

Fundamental Limit on Size of Load that Can be Driven at given Clock Rate



Can $C_1 = 100 pF$ be clocked at 500 MHz if a pad driver is used?

$$f_{IN-MAX} = 1/t_{PROP}$$

Assume C_{REF}=4fF,
$$t_{REF}$$
=20ps f_{IN-MAX} =1/ t_{PROP} $FI_{LOAD} = \frac{100pF}{4fF} = 25,000$

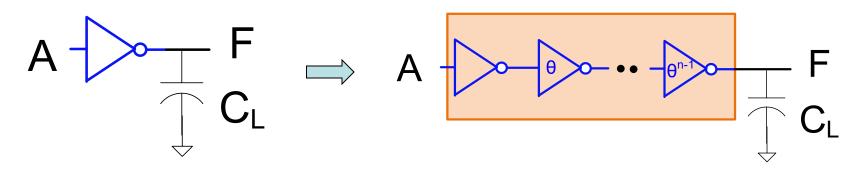
$$n_{OPT} = ln \left(\frac{C_L}{C_{REF}} \right) = ln(FI_L) = 10.1 \approx 10$$

$$t_{PROP} = n\theta t_{REF} = 10 \bullet e \bullet t_{REF} = 542 \text{ psec}$$

$$f_{IN-MAX} = \frac{1}{n\theta t_{REF}} = \frac{1}{542 \text{ psec}} = 1.85 \text{ GHz}$$

Yes!

Fundamental Limit on Size of Load that Can be Driven at given Clock Rate



Can $C_1 = 500pF$ be clocked at 2GHz if a pad driver is used?

$$f_{IN-MAX} = 1/t_{PROP}$$

Assume
$$C_{REF} = 4fF$$
, $t_{REF} = 20ps$ $f_{IN-MAX} = 1/t_{PROP}$ $FI_{LOAD} = \frac{500pF}{4fF} = 125,000$

$$n_{OPT} = In \left(\frac{C_L}{C_{REF}} \right) = In(FI_L) = 11.7 \approx 12$$

$$t_{PROP} = n\theta t_{REF} = 12 \bullet e \bullet t_{REF} = 652 \text{ psec}$$

$$f_{IN-MAX} = \frac{1}{n\theta t_{REF}} = \frac{1}{652 \text{ psec}} = 1.54 \text{ GHz}$$

No!

Digital Circuit Design

- Hierarchical Design
- Basic Logic Gates
- Properties of Logic Families
- Characterization of CMOS Inverter
- Static CMOS Logic Gates
 - Ratio Logic
- Propagation Delay
 - Simple analytical models
 - FI/OD
 - Logical Effort
 - Elmore Delay
 - Sizing of Gates
 - The Reference Inverter

- Propagation Delay with Multiple Levels of Logic
- Optimal driving of Large Capacitive Loads
- Power Dissipation in Logic Circuits
 - Other Logic Styles
 - Array Logic
 - Ring Oscillators

done

partia



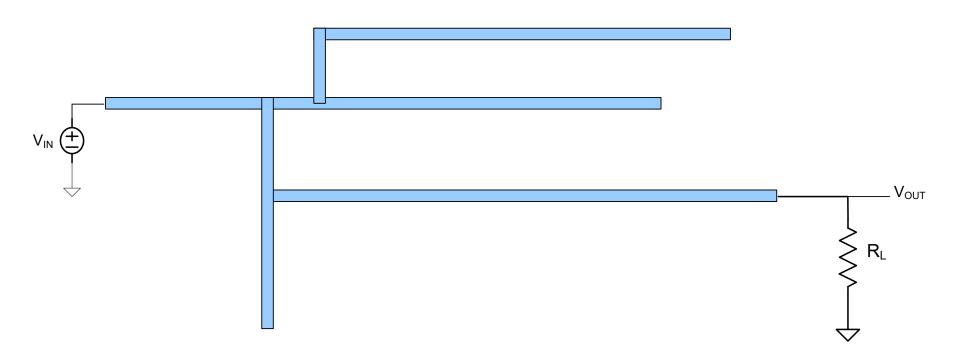


- Interconnects have a distributed resistance and a distributed capacitance

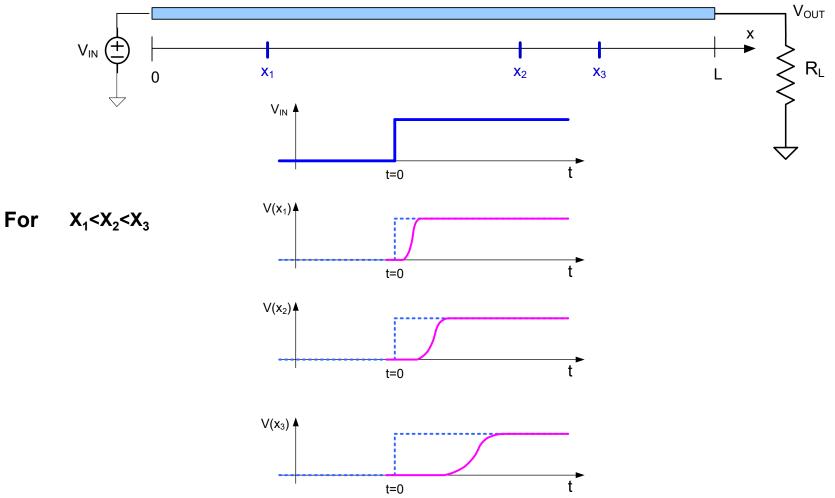
 Often modeled as resistance/unit length and capacitance per unit length
- These delay the propagation of the signal
- Effectively a transmission line
 - analysis is really complicated
- Can have much more complicated geometries



Can have much more complicated geometries



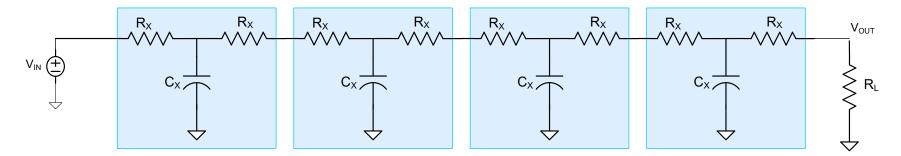








A lumped element model of transmission line (with "T" elements)

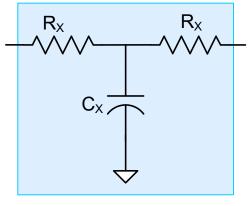


Even this lumped model is 4-th order and a closed-form solution is very tedious!

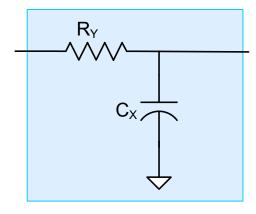
Can use "L" or other lumped segments as well (with small number some perform better than others)

Need a quick (and reasonably good) approximation to the delay of a delay line !!

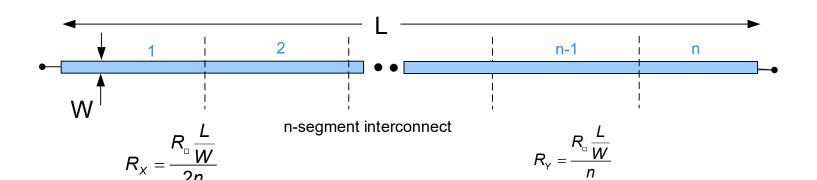




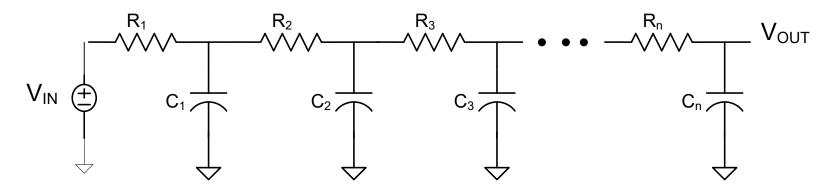
T-Model



L-Model







Elmore delay:
$$t_{ED} = \sum_{i=1}^{n} \left(C_i \sum_{j=1}^{i} R_j \right)$$

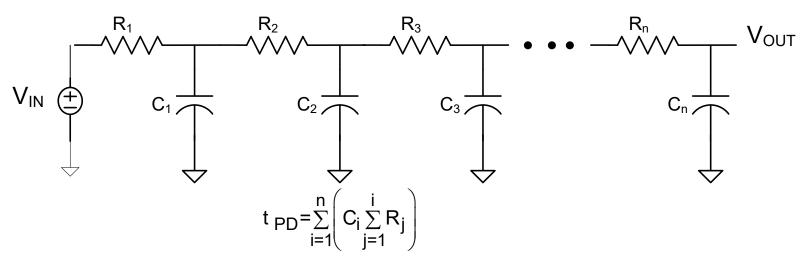
- It can be shown that this is a reasonably good approximation to the actual delay
 - provided sufficient number of stages are used
 - number does not need to be very large
- Numbering is critical (resistors and capacitors numbered from input to output)
- As stated, only applies to this specific structure
- HL and LH Elmore Delays are the same
- Since t_{EHL}=t_{ELH}, t_{PROP} = 2 t_{ED}

Elmore delay:
$$t_{PD} = \sum_{i=1}^{n} \left(C_i \sum_{j=1}^{i} R_j \right)$$

Note error in text on Page 161 of first edition of WH

$$t_{pd} = \sum_{i} R_{n-i} C_{i} = \sum_{i=1}^{N} C_{i} \sum_{j=i}^{i} R_{j}$$

Not detailed definition on Page 150 of second edition of WH



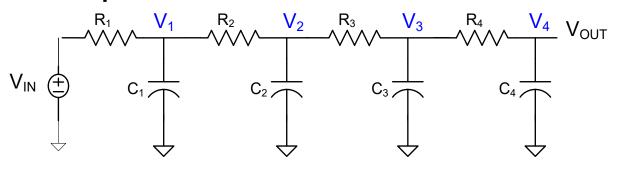
From Wikipedia (Dec 8 2021):

Elmore delay[1] is a simple approximation to the delay through an RC network in an electronic system. It is often used in applications such as logic synthesis, delay calculation, static timing analysis, placement and routing, since it is simple to compute (especially in tree structured networks, which are the vast majority of signal nets within ICs) and is reasonably accurate. Even where it is not accurate, it is usually faithful, in the sense that reducing the Elmore delay will almost always reduce the true delay, so it is still useful in optimization. [1] W.C. Elmore. The Transient Analysis of Damped Linear Networks with Particular Regard to 22

Wideband Amplifiers. J. Applied Physics, vol. 19(1), 1948.



Example:



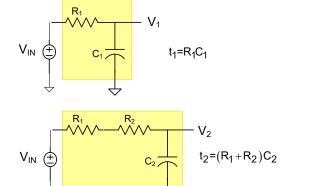
Elmore delay:

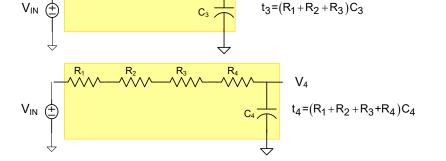
$$t_{ED} = \sum_{i=1}^{4} \left(C_i \sum_{j=1}^{i} R_j \right)$$

$$t_{ED} = \sum_{i=1}^{4} \left(t_i \right)$$
where
$$t_i = C_i \sum_{j=1}^{i} R_j \qquad j = 1, 2, 3, 4$$

What is really happening?

- Creating 4 first-order circuits
- Delay to V₁, V₂, V₃ and V₄ calculated separately by considering capacitors one at a time and assuming others are 0

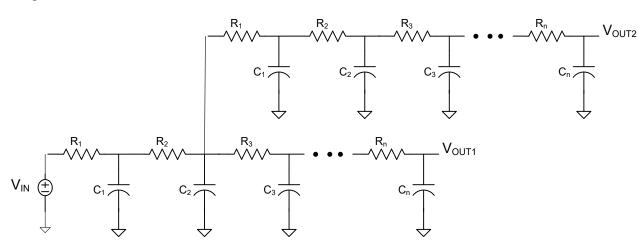




Extensions:

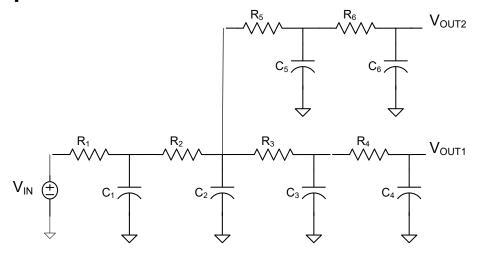


Lumped Network Model:

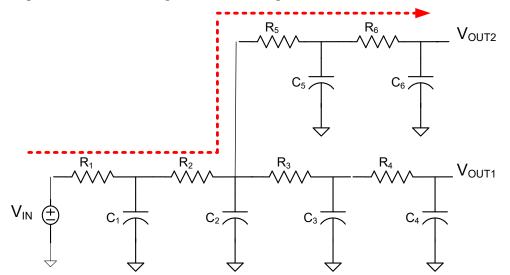


Extensions:

1. Create a lumped element model

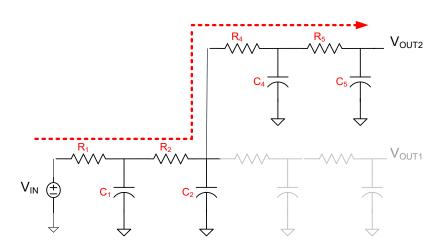


2. Identify the path from input to output



Extensions:

3. Renumber elements along path from input to output and neglect off-path elements



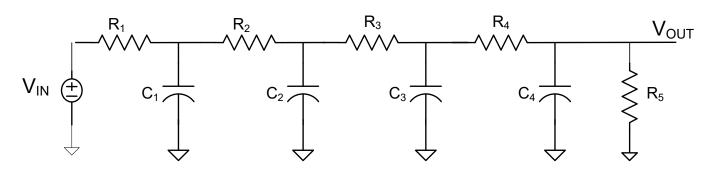
4. Use Elmore Delay equation for elements on this RC network

$$t_{ED} = \sum_{i=1}^{4} \left(C_i \sum_{j=1}^{i} R_j \right)$$



How is a resistive load handled?

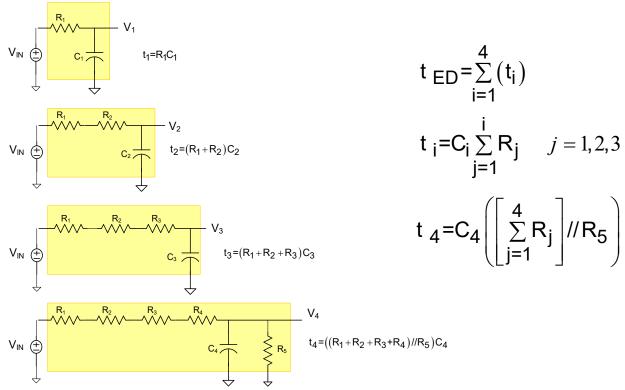
Example with resistive load:



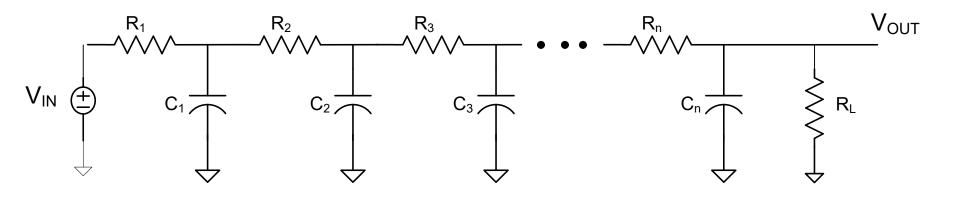
Elmore delay:

$$t_{ED} = \sum_{i=1}^{4} \left(C_i \sum_{j=1}^{i} R_j \right)$$

where



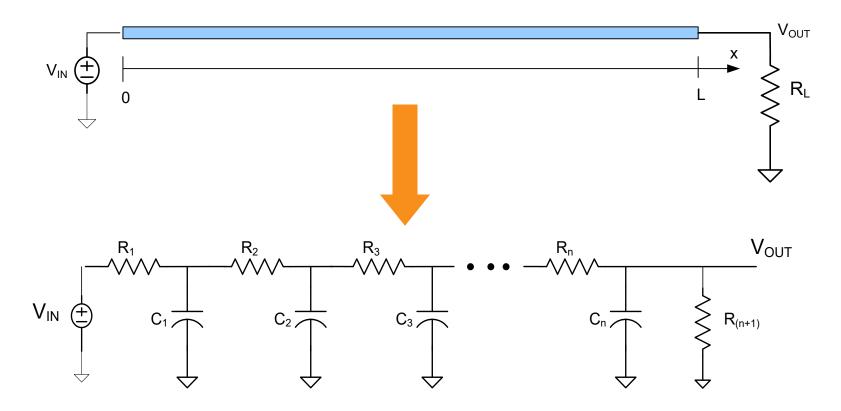
With resistive load:



Simple Elmore delay:

$$t_{ED} = \sum_{i=1}^{n-1} \left(C_i \sum_{j=1}^{i} R_j \right) + C_n \left(\left(\sum_{j=1}^{n} R_j \right) / / R_L \right)$$

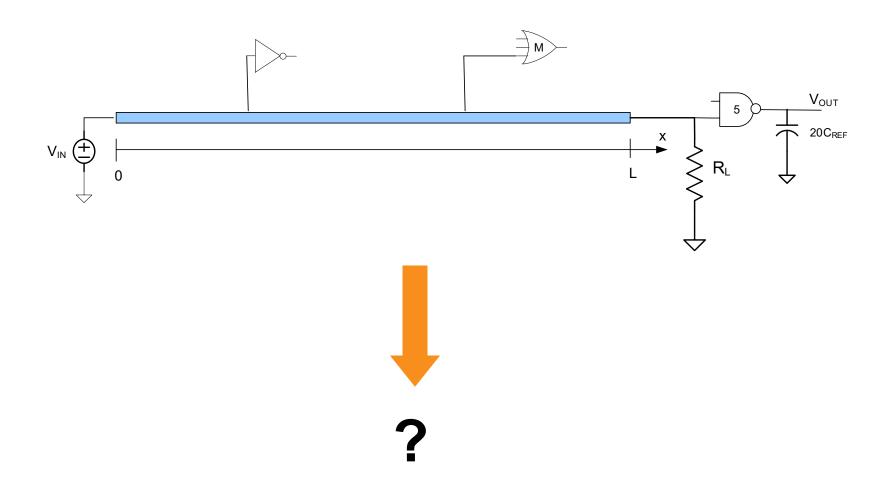
Actually, R_L affects all of the delays and a modestly better but modestly more complicated delay model is often used



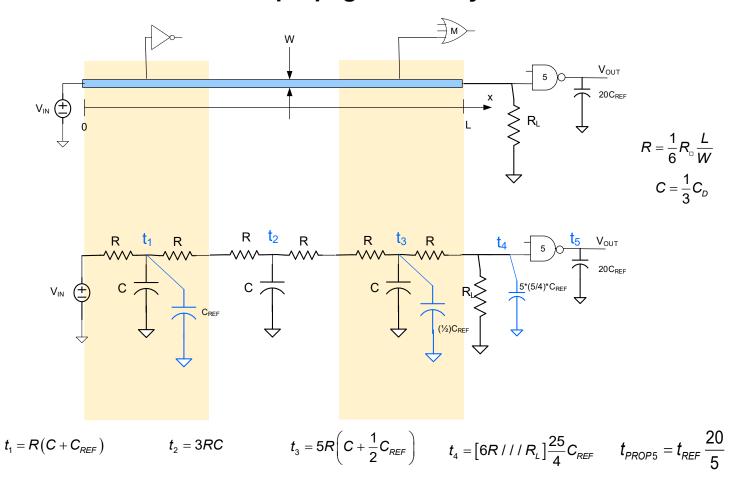
How are the number of stages chosen?

- For hand analysis, keep number of stages small (maybe 3 or 4 for simple delay line) if possible)
- If "faithfulness" is important, should keep the number of stages per unit length constant

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Determine propagation delay



$$t_{PROP} = 2\sum_{i=1}^{4} t_i + t_{PROP5}$$

Digital Circuit Design

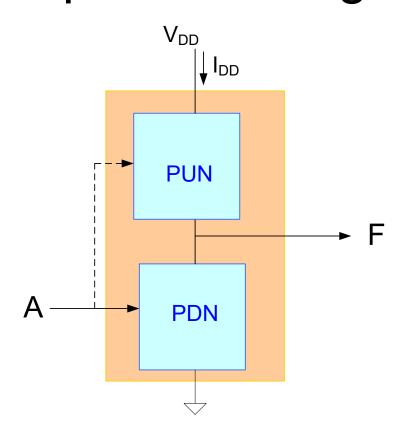
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Power Dissipation in Logic Circuits



Assume current periodic with period T_{CL}

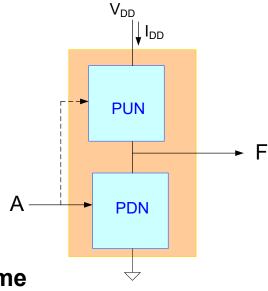
$$P_{AVG,T} = \frac{1}{T_{CL}} \int_{t_1}^{t_1 + T_{CL}} V_{DD} I_{DD}(t) dt$$

Power Dissipation in Logic Circuits

Types of Power Dissipation

- Static
- Pipe
- Dynamic
- Leakage
 - Gate
 - Diffusion
 - Drain

Static Power Dissipation



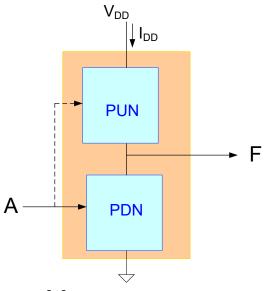
If Boolean output averages H and L 50% of the time

$$P_{STAT,AVG} = \frac{P_{H} + P_{L}}{2}$$

$$P_{STAT,AVG} = \frac{V_{DD}(I_{DDH} + I_{DDL})}{2}$$

- Generally decreases with V_{DD}
- I_{DDH}=I_{DDL}=0 for static CMOS gates so P_{STAT}=0
- A major source of power dissipation in ratio logic circuits and the major reason CMOS is so widely used

Pipe Power Dissipation

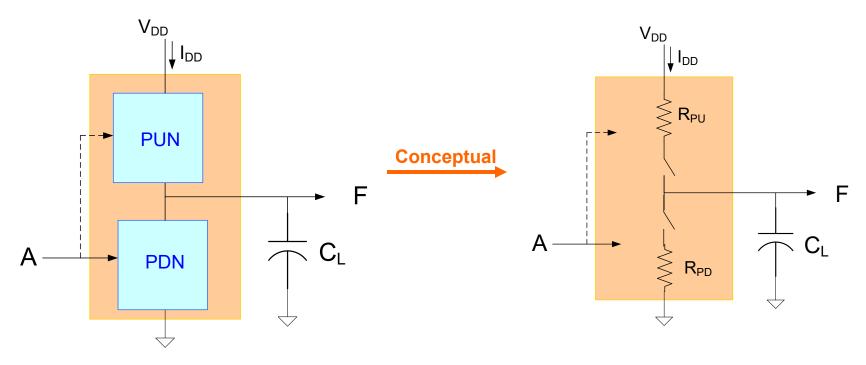


Due to conduction of both PUN and PDN during transitions

- Can be made small if transitions are fast
- Usually negligible in Static CMOS circuits



Dynamic Power Dissipation



Due to charging and discharging C₁ on logic transitions

 \mathbf{C}_{L} dissipates no power but PUN and PDN dissipate power during charge and discharge of \mathbf{C}_{L}

C₁ includes all gate input capacitances of loads and interconnect capacitances



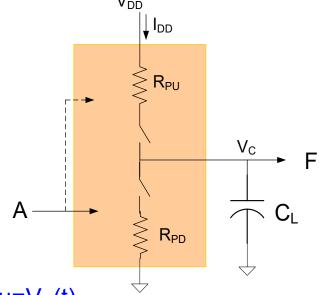
Energy supplied by V_{DD} when C_L charges

Assume a HL input transition starts at t = t₁

$$E = \int_{1}^{\infty} V_{DD}I_{DD}(t)dt$$

$$t_{1}$$

$$I_{DD} = C_{L} \frac{dV_{C}}{dt}$$



$$E = \int_{t_1}^{\infty} V_{DD}C_L \frac{dV_C}{dt} dt$$
 change variable $u=V_C(t)$

$$E = \int\limits_{V_C=0}^{V_{DD}} V_{DD} C_L dV_C = V_{DD} C_L \int\limits_{V_C=0}^{V_{DD}} dV_C = V_{DD} C_L \left. V_C \right|_{V_C=0}^{V_{DD}} = V_{DD}^2 C_L \left. V_C \right|_{V_C=0}^{V_$$

Energy stored in C_1 after C_1 is charged to V_{DD} :

$$E = \frac{1}{2}C_L V_{DD}^2$$



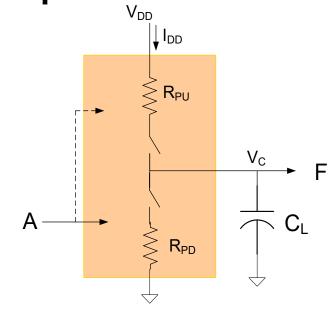
Energy supplied by V_{DD} and dissipated in R_{PU} when C_{I} charges

$$E_{DIS} = \frac{1}{2}C_L V_{DD}^2$$

Energy stored on C_L after L-H transition

$$E_{STORE} = \frac{1}{2}C_L V_{DD}^2$$

$$E = E_{DIS} + E_{STORE} = C_L V_{DD}^2$$



When the output transitions from H to L, energy stored on C_L is dissipated in PDN

Thus, energy from V_{DD} for one L-H: H-L output transition sequence is

$$E = C_L V_{DD}^2$$



Dynamic Power Dissipation

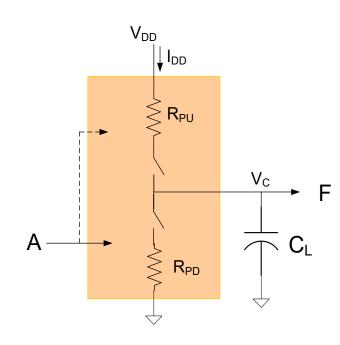
Energy from V_{DD} for one L-H: H-L output transition sequence is

$$E=C_LV_{DD}^2$$

If f is the average transition rate of the output, determine P_{AVG}

$$P_{AVG} = \frac{E}{T} = Ef$$

$$P_{DYN} = fC_L V_{DD}^2$$



If a gate has a transition duty cycle of 50% with a clock frequency of fcl

$$P_{DYN} = \frac{f_{CL}}{2} C_L V_{DD}^2$$

Note dependent on the square of V_{DD} ! Want to make V_{DD} small !!!

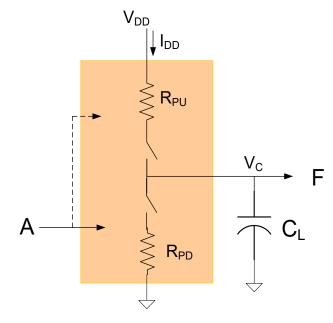
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Dynamic Power Dissipation

Energy dissipated with clock signal itself

$$P_{DYN} = f_{CL}C_LV_{DD}^2$$



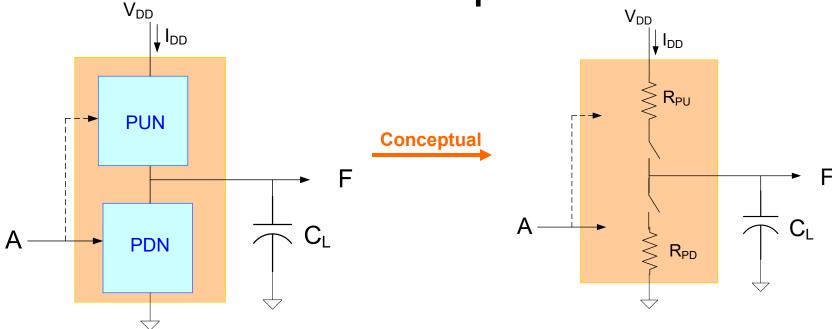
The clock transitions on every clock cycle (i.e. it has a transition duty cycle of 100%)

Clock distribution can cause significant power dissipation

But if a gate has a transition duty cycle of 50% with a clock frequency of f_{CL}

$$P_{DYN} = \frac{f_{CL}}{2} C_L V_{DD}^2$$

Power Dissipation



- All power is dissipated in pull-up and pull-down devices
- C₁ dissipates no power but PUN and PDN dissipate power when charging and discharging C₁
- Dynamic power dissipation reduced by more (often much more) than a factor of 2 if minimum sizing strategy is used
- NAND logic more attractive than NOR logic when multiple inputs ⁴⁵ required



Stay Safe and Stay Healthy!

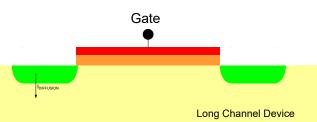
End of Lecture 43



Leakage Power Dissipation

- Gate

- with very thin gate oxides, some gate leakage current flows
- major concern in 60nm and smaller processes
- actually a type of static power dissipation

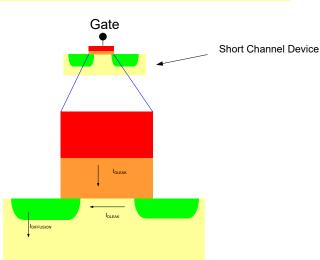


-Diffusion

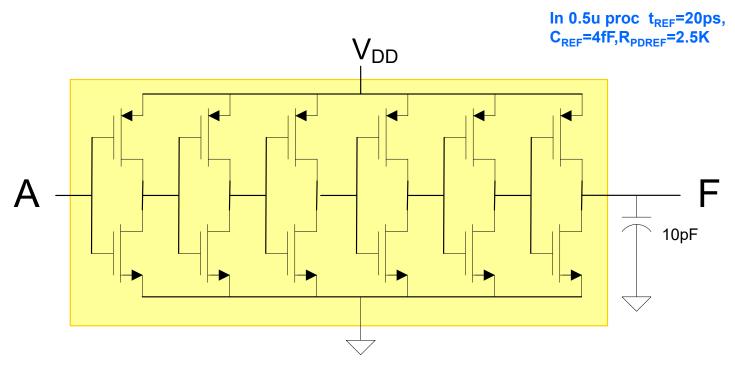
- Leakage across a reverse-biased pn junction
- Dependent upon total diffusion area
- May actually be dominant power loss on longerchannel devices
- Actually a type of static power dissipation

-Drain

- channel current due to small V_{GS}-V_T
- of significant concern only with low V_{DD} processes
- actually a type of static power dissipation



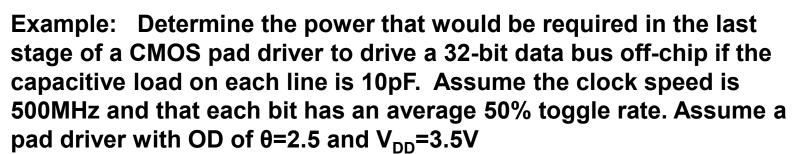
Example: Determine the dynamic power dissipation in the last stage of a 6-stage CMOS pad driver if used to drive a 10pF capacitive load if the system clock is 500MHz and the output changes with 50% of the clock transitions. Assume pad driver with OD of θ =2.5 and V_{DD} =3.5V



Solution: (assume output changes with 50% of clock transitions)

$$P_{DYN} = \frac{f_{CL}}{2} C_L V_{DD}^2 = \frac{5E8}{2} \cdot 10 pF \cdot 3.5^2 = 30.5 mW$$

Note this solution is independent of the OD and the process



A F 10pF

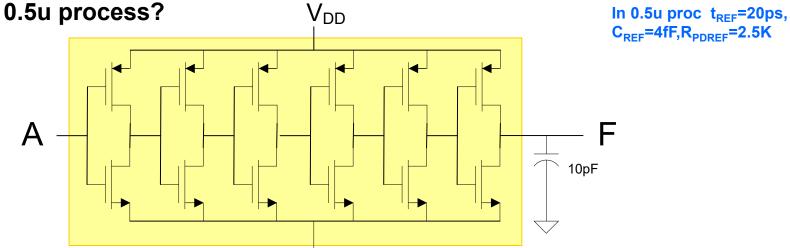
In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

Solution:

$$P_{DYN} = 32 \cdot \frac{f_{CL}}{2} C_L V_{DD}^2 = 32 \cdot \frac{5E8}{2} \cdot 10 pF \cdot 3.5^2 = 980 mW$$

Note: A very large amount of power is required to take a large bus off-chip if bus has a high rate of activity.

Example: Will the CMOS pad driver actually be able to drive the 10pF load with a system clock of 500MHz as in the previous example in the

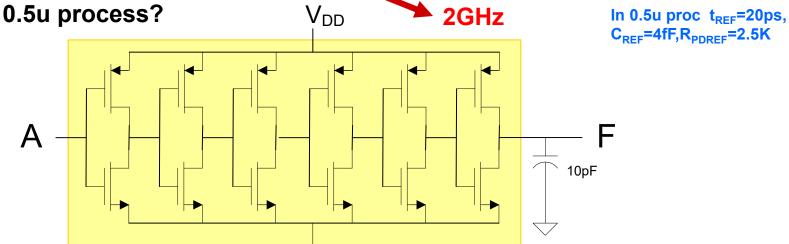


Solution – since outputs are data dependent, output must be able to operate 500Mz:

$$t_{CLK} = \frac{1}{500 \text{MHz}} = 2 \text{nsec}$$
 $FI_{load} = \frac{10 pF}{4 fF} = 2500$ $OD_6 = \theta^5 = 98$ $t_{PROP} = 5\theta \bullet t_{REF} + \frac{FI_{load}}{OD_6} t_{REF}$ $\frac{FI_{load}}{OD_6} = \frac{2500}{98} \cong 25$

 $t_{prop} = 5 \cdot 2.5 \cdot 20psec + 25 \cdot 20psec = (12.5 + 25)20psec = 0.75nsec$

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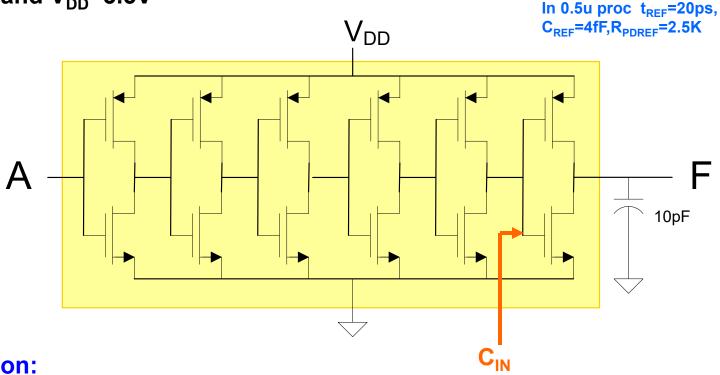
since t_{CLK} t_{PROP}, this pad driver can drive the 10pF load at 500MHz.

t_{CLK} < t_{PROP}

can not



Example: Determine the dynamic power dissipation in the <u>next to the last stage</u> of a 6-stage CMOS pad driver if used to drive a 10pF capacitive load if clocked at 500MHz. Assume pad driver with OD of θ =2.5 and V_{DD} =3.5V

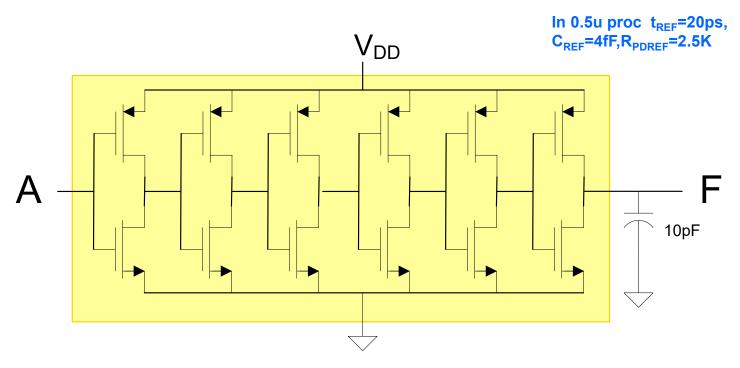


Solution:

$$C_{IN} = \theta^5 C_{REF} = 2.5^5 \cdot 4 fF = 390 fF$$

$$P_{DYN} = f_{CL}C_LV_{DD}^2 = 5E8 \cdot 390 fF \cdot 3.5^2 = 2.4 mW$$

Example: Is the 6-stage CMOS pad driver adequate to drive the 10pF capacitive load as fast as possible? Assume pad driver with OD of θ =2.5 and V_{DD} =3.5V



Solution:

$$n_{OPT} = ln \left(\frac{C_L}{C_{REF}} \right) = ln \left(\frac{10pF}{4fF} \right) = 7.8$$

No – an 8-stage pad driver would drive the load much faster (but is not needed If clocked at only 500MHz)



Stay Safe and Stay Healthy!

End of Lecture 43